LS-75
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Storing Electrons Instead of Positrons Without Trapping of Positive Ions

In this note, a procedure to inject and store electrons in the ring without trapping ions will be described. The minimum injection current per bunch below which ion trapping occur will be determined. Since the vertical beam size is smaller than the horizontal beam size, we will consider the vertical motion only.

The following assumptions are made:

1. The bunch period is much longer than the bunch length. This assumption allow us to use the thin lens approximation for the focusing effect of the electron bunch.

2. The bunch length is much larger than the transverse dimensions of the beam. A two-dimensional calculation can then be used to obtain the focusing field of the electron bunch.

   Assume a bunch of electrons is injected in the storage ring. Initially, the number of ions is small and the field generated by the ions can be neglected. In the thin lens approximation, the motion of the ions can be written in the form

$$\begin{pmatrix} Y \\ \dot{Y} \end{pmatrix}_{out} = \begin{pmatrix} 1 - \tau/2F & \tau(1 - \tau/4F) \\ - 1/F & 1 - \tau/2F \end{pmatrix} \begin{pmatrix} Y \\ \dot{Y} \end{pmatrix}_{in} . \tag{1}$$

Here τ is the bunch period and F is the focal length given by

$$\frac{1}{F} = \frac{n_e \tau_1 e^2}{\varepsilon_o M} \frac{a}{a+b} , \tag{2}$$

where τ_1 = bunch length

n_e = electron density

$2a$ = horizontal beam size

$2b$ = vertical beam size

M = ion mass

From Eq. (1), we see that in order for the motion to be unstable, it is necessary and sufficient that

$$\cos\sigma = 1 - \tau/2F < -1 \ ,$$

or using Eq. (2)

$$\frac{n_e \ \tau_1 \ e^2 \ \tau}{\varepsilon_o \ M} \ \frac{a}{a+b} > 4 \ .$$

Introducing the classical electron radius r_e given by $r_e = \dfrac{e^2}{4\pi\varepsilon_o \ mc^2}$ and setting $n_e \ e \ \tau_1 = \dfrac{I_b \ 2\pi R}{\pi ab \ c^2}$, where I_b is the average current per bunch, we obtain

$$\frac{I_b}{I_a} \ \frac{m}{M} \ \frac{2\pi R}{b(a+b)} \ c\tau > 1 \ . \tag{3}$$

Here $I_a = \dfrac{ec}{r_e} \approx 1.7 \times 10^4$ A .

The quantities a and b are determined by the output emittance of the booster

$$\varepsilon_x = 0.132 \text{ mm-mrad}$$
$$\varepsilon_y = 0.0132 \text{ mm-mrad.}$$
(10% coupling)

For the CDR storage ring, we have $\bar{\beta}_x \approx \bar{\beta}_y \approx 10$ m and obtain

$$a = \sqrt{2\bar{\beta}_x \varepsilon_x} \approx 1.6 \text{ mm}$$

$$b = \sqrt{2 \ \bar{\beta}_y \ \varepsilon_y} \approx 0.5 \text{ mm}$$

In a high vacuum system, the heaviest molecule of reasonable abundance is CO_2 (~1% partial pressure, $\frac{M}{m} \approx 8 \times 10^4$) . Substitution in Eq. (3) of a = 1.6 mm, b = 0.5 mm, $I_a = 1.7 \times 10^4$ A, $\frac{M}{m}$ 8×10^4 and $2\pi R = 800$ m give $I_b > 2.23$ mA, or using the relation I = Nef with $f = 3.75 \times 10^5$ Hz, we find $N > 3.72 \times 10^{10}$. Thus, the booster must be able to deliver 3.72×10^{10} electrons in a single bunch.

It is not difficult to see from Eq. (3) that, as the beam size decreases due to radiation damping, the motion of the ions become more unstable. Furthermore, it can be shown that no trapping of ions will occur if the designated buckets of the storage ring are filled completely one at a time and sequentially. Setting in Eq. (3) $c\tau = \frac{2\pi R}{m_b}$, where m_b is the number of bunches, we obtain

$$I_b > m_b \frac{b(a+b)}{(2\pi R)^2} \frac{M}{m} I_a \; . \tag{4}$$

This equation gives the minimum average bunch current as a function of the number of bunches. For the CDR storage ring,

$\varepsilon_x = 7 \times 10^{-3}$ mm-mrad

$\varepsilon_y = 7 \times 10^{-4}$ mm-mrad (10% coupling)

so that $a = \sqrt{2\bar{\beta}_x \varepsilon_x} \approx 0.37$ mm and

$b = \sqrt{2\bar{\beta}_y \varepsilon_y} \approx 0.12$ mm .

Substitutions of these values of a and b in Eq. (4) give for $2\pi R = 800$ m,

$\frac{M}{m} = 8 \times 10^4$ and $I_a = 1.7 \times 10^4$ A, $I_b > 0.125\, m_b$ mA.

If during the storage, due to instabilities, the current per bunch is reduced to below this value, ions will be trapped, which may result in a larger beam size.

Example

Storage of 200 mA electrons in 36 bunches

Linac:

Energy	200 MeV
Current	0.6 A
Bunch length	16.5 nsec
Energy spread	$<$ ±3 MeV
Repetition rate	1 Hz

Two booster bunches per storage ring bunch

Filling time	$\approx$ 72 sec